Query Answering in Data Integration

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Outline

1. Quick reminder
2. Computing certain answers under OWA/CWA
3. Inverse rules algorithm
4. MiniCon algorithm
5. Coping with integrity constraints and access patterns
6. Rewriting using views in presence of access patterns, integrity constraints, disjunction and negation
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Quick reminder

Data integration

- global relations (mediated schema)—used in queries
- source relations—store actual data,
- mapping: LAV—each source relation described as a result of a query over the global relations,
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- source relations—store actual data, view instance \( I \),
- mapping: LAV—each source relation described as a result of a query over the global relations, view definitions \( \mathcal{V} = (V_1, \ldots, V_n) \),

Certain answers
- certain answers for \( Q \)—a set of tuples \( Q(D) \) for each database \( D \) consistent with a given instance of source relations,
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Certain answers

- certain answers for \( Q \)—a set of tuples \( Q(D) \) for each database \( D \) consistent with a given instance of source relations,
- \( t \) is a certain answer
  - under OWA (views are sound) if \( t \) is an element of \( Q(D) \) for each database \( D \) such that \( I \subseteq \mathcal{V}(D) \)
  - under CWA (views are exact) if \( t \) is an element of \( Q(D) \) for each database \( D \) such that \( I = \mathcal{V}(D) \)
## Quick reminder

### Data integration

- global relations (mediated schema)—used in queries
- source relations—store actual data, **view instance** $I$
- mapping: LAV—each source relation described as a result of a query over the global relations, **view definitions** $\mathcal{V} = (V_1, \ldots, V_n)$

### Query rewriting

- query rewriting using views—mentions the source relations only, can be equivalent or maximally-contained (possibly relative to a set of constraints).
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Query Answering vs. Incomplete Databases

**Idea**
- Views (=source data) represent many possible (global) databases
- Idea: use techniques in incomplete databases

**Example**

View definitions:

\[ v(0, Y) : \neg p(0, Y) \]
\[ v(X, Y) : \neg p(X, Z), p(Z, Y) \]

View instance:

\{ v(0, 1), v(1, 1) \}
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**Conditional table (OWA):**

<table>
<thead>
<tr>
<th>p:</th>
<th>0</th>
<th>1</th>
<th>w = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 x</td>
<td>w \neq 1</td>
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<td>x 1</td>
<td>w \neq 1</td>
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<td>1 u</td>
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### Idea
- Views (=source data) represent many possible (global) databases
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### Example

#### View definitions:

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<tr>
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#### View instance:

| View instance       | $(v(0, 1), v(1, 1))$                           |

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#### Conditional table (CWA):

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Simple reductions between the two problems in both directions exist (for views and queries in CQ, CQ\(\neq\), PQ, datalog)

Reduction to query containment

Input: \(\mathcal{V} = (v_1, \ldots, v_k), Q, I\) and a tuple \(t\).
Let \(Q'\) be the query consisting of all the definitions \(\mathcal{V}\) together with:

\[
q'(t) : = \quad v_1(t_{i1}), \ldots, v_1(t_{in_1}), \ldots \\
\quad \quad \quad v_1(t_{k1}), \ldots, v_k(t_{kn_1})
\]

where \(I(v_i) = \{t_{i1}, \ldots, t_{in_i}\}\)
Then \(t\) is a certain answer iff \(Q' \subseteq Q\).
Simple reductions between the two problems in both directions exist (for views and queries in CQ, CQ\(\neq\), PQ, datalog)

### Reduction to computing certain answers

Input: \(Q_1\) and \(Q_2\).

Let the view definition be the rules of \(Q_1\) together with

\[
v(c) : - \ q_1(X), p(X)
\]

Let the instance \(I = \{v(c)\}\) and let \(Q\) consists of all the rules of \(Q_2\) together with

\[
q(c) : - \ q_2(X), p(X)
\]

Then \(Q_1 \subseteq Q_2\) iff \((c)\) is a certain answer.
Simple reductions between the two problems in both directions exist (for views and queries in CQ, CQ≠, PQ, datalog)

Consequences
Decidability and undecidability results carry over in both directions. If the problems are decidable then the **combined complexity** of computing certain answers is the same as the **query complexity** of query containment.
Data complexity of computing certain answers under OWA

<table>
<thead>
<tr>
<th>views</th>
<th>query</th>
<th>CQ</th>
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Maximally contained rewriting vs. certain answers

- A datalog query $\mathcal{P}$ is a **query plan** if all EDB predicates in $\mathcal{P}$ are views literals.

- The **expansion** $\mathcal{P}^{\text{exp}}$ of a query plan $\mathcal{P}$ is $\mathcal{P}$ with all views literals replaced with their definitions.

- A query plan $\mathcal{P}$ is **maximally-contained** in a datalog query $Q$ w.r.t. view definitions $V$ if
  - $\mathcal{P}^{\text{exp}} \subseteq Q$, and
  - for each query plan $\mathcal{P}'$ with $(\mathcal{P}')^{\text{exp}} \subseteq Q$ we have $(\mathcal{P}')^{\text{exp}} \subseteq \mathcal{P}^{\text{exp}}$. 
Maximally contained rewriting vs. certain answers

Theorem

Let $\mathcal{V} \subseteq CQ$, $Q \in \text{datalog}$, let $\mathcal{P}$ be maximally-contained in $Q$ w.r.t. $\mathcal{V}$. Then for each view instance $\mathcal{I}$ the query plan $\mathcal{P}$ computes exactly the certain answers of $Q$ under OWA.

Proof.

$I$ - view instance such that $\mathcal{P}$ fails to compute a certain answer $t$. $\mathcal{P}'$ - the query plan $\mathcal{P}$ with two additional rules:

$$
\begin{align*}
  r_1 & : \quad q'(X) : - q(X) \\
  r_2 & : \quad q'(t) : - v_1(t_{11}), \ldots, v_1(t_{1n_1}), \ldots, v_1(t_{k1}), \ldots, v_k(t_{kn_1})
\end{align*}
$$

where $I(v_i) = \{t_i, \ldots, t_{in_i}\}$ and $q$ is the answer predicate of $\mathcal{P}$.

$(\mathcal{P}')^\text{exp}$ is contained in $Q$ but it is not contained in $(\mathcal{P})^\text{exp}$. That contradicts the maximal containment of $\mathcal{P}$ in $Q$. \qed
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Inverse rules

Example

Data sources

\[ s_1(X, Y) : – \text{edge}(X, Z), \text{edge}(Z, W), \text{edge}(W, Y) \]
\[ s_2(X) : – \text{edge}(X, Z) \]
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\[ \text{edge}(X, f_1(X, Y)) : - s_1(X, Y) \]

The fresh function symbol \( f_{r,i} \)
for each rule \( r \) and each existential variable \( X_i \) in \( r \)
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\end{align*}
\]

Inverse rules

\[
\begin{align*}
  \text{edge}(X, f_1(X, Y)) & : - \quad s_1(X, Y) \\
  \text{edge}(f_1(X, Y), f_2(X, Y)) & : - \quad s_1(X, Y) \\
  \text{edge}(f_1(X, Y), Y) & : - \quad s_1(X, Y) \\
  \text{edge}(X, f_3(X)) & : - \quad s_2(X)
\end{align*}
\]

The fresh function symbol \( f_{r,i} \)

for each rule \( r \) and each existential variable \( X_i \) in \( r \)
Inverse rules algorithm (1)

Example

Query $Q$:

\[
q(X, Y) : - \text{edge}(X, Y)
\]

\[
q(X, Y) : - \text{edge}(X, Z), \text{edge}(Z, Y)
\]
## Inverse rules algorithm (1)

**Example**

**Query** $Q$:

\[
\begin{align*}
q(X, Y) & : \neg \text{edge}(X, Y) \\
q(X, Y) & : \neg \text{edge}(X, Z), \text{edge}(Z, Y)
\end{align*}
\]

**Data source**:

\[
\begin{align*}
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\end{align*}
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Inverse rules algorithm (1)

Example

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\]

Data source:

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\begin{align*}
s(X, Y) & : - \quad \text{edge}(X, Z), \text{edge}(Z, Y)
\end{align*}
\]

Query plan \((Q, V^{-1})\):

\[
\begin{align*}
q(X, Y) & : - \quad \text{edge}(X, Y) \\
q(X, Y) & : - \quad \text{edge}(X, Z), \text{edge}(Z, Y) \\
\text{edge}(X, f(X, Y)) & : - \quad s(X, Y) \\
\text{edge}(f(X, Y), Y) & : - \quad s(X, Y)
\end{align*}
\]
### Example

**Query \( Q \):**

No longer datalog, but we can evaluate it in two stages:

- start with the inverse rules (they introduce function symbols but are not recursive),
- apply the rules of \( Q \), (they are recursive but do not introduce function symbols).

In fact, with a little bit of bureaucracy we can get rid of function symbols at all.

\[
\begin{align*}
\text{edge}(X, t(X, Y)) & : - \quad s(X, Y) \\
\text{edge}(f(X, Y), Y) & : - \quad s(X, Y)
\end{align*}
\]
Inverse rules algorithm (2)

Inverse rules algorithm

- Compute plan \((Q, V^{-1})\downarrow\) that returns the same set of tuples as \((Q, V^{-1})\) but filters out the tuples that contain function symbol(s).
- Evaluate \((Q, V^{-1})\downarrow\) on a set of data sources.
Applying inverse rules is like chasing $\mathcal{I}$ with the view definitions.

Example

\[
\begin{align*}
  s(X, Y) : & \quad \neg \text{edge}(X, Z), \text{edge}(Z, Y) \\
  \forall X, Y \ s(X, Y) : & \quad \exists Z \ \text{edge}(X, Z), \text{edge}(Z, Y)
\end{align*}
\]

\[
\begin{array}{c|c|c}
  \mathcal{I}: & s: & 0 & 0 \\
  0 & 1 & \\
  3 & 2 & \\
\end{array}
\quad
\begin{array}{c|c|c}
  \mathcal{V}^{-1}(\mathcal{I}): & \text{edge}: & 0 & Z_1 \\
  & & Z_1 & 0 \\
  & & 0 & Z_2 \\
  & & Z_2 & 1 \\
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Problems with the inverse rules algorithm

The inverse rules algorithm produces expensive query plans
- Does not use views directly, forces a lot of recomputation.
  e.g. \( Q(\bar{x}) = V(\bar{x}) \)
- May compute useless tuples i.e. may invert the extensions of the views that are not needed in the rewriting.
Another approach: bucket algorithm

1. Create buckets, one for each subgoal \( g \) in \( Q \). The bucket for \( g \) contains the views with subgoals to which \( g \) can be mapped.

2. For each element of the Cartesian product of the buckets
   1. construct a conjunctive rewriting \( r \),
   2. check the containment of \( r \) in \( Q \)
      (equate some pairs of variables in \( r \), if necessary).

Example

\[
\begin{align*}
q(X) & : - e(X, Y), e(Y, X), p(X, Y) \\
v_1(U) & : - e(U, V), e(V, U) \\
v_2(U, V) & : - p(U, V) \\
v_3(U, W) & : - e(U, V), e(V, W), p(U, V)
\end{align*}
\]

Rewritings

\[
\begin{align*}
q_0(X) & : - v_1(X), v_2(X, Y) \\
& \ldots \\
q_i(X) & : - v_3(X, X)
\end{align*}
\]
Another approach: bucket algorithm

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2. For each element of the Cartesian product of the buckets
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### Example

- $q(X) : - e(X, Y), e(Y, X), p(X, Y)$
- $v_1(U) : - e(U, V), e(V, U)$
- $v_2(U, V) : - p(U, V)$
- $v_3(U, W) : - e(U, V), e(V, W), p(U, V)$

### Problems

- Expensive - tries many useless combinations,
- e.g. $v_1$ useless in the rewriting: since $Y$ is not distinguished it is not possible to join $e(X, Y)$ and $p(X, Y)$. 
MiniCon algorithm

Better idea
- As before, for each subgoal $g$ in $Q$ find the views with subgoals to which $g$ can be mapped.
- But then, given such a partial mapping, finds minimal additional set of subgoals in the query that have to be mapped together.

MiniCon Descriptions (MCDs)
MCD $C$ for a query $Q$ over a view $v$ consists of
- head homomorphism $h$ (may equate head variables e.g. $v_3(X, X)$),
- partial mapping $\varphi$ from $\text{Vars}(Q)$ to $\text{Vars}(V)$.
- some subset $G$ of subgoals in $Q$ that are covered by some subgoal in $h(V)$ using $\varphi$. 
MCDs

Key Property

MCD C for Q over V can only be used in a non-redundant rewriting of Q if:

C1 For each head variable x of Q in domain of ϕ, ϕ(x) is head variable in MCD view (i.e. in h(V)).

C2 If a variable participates in a join predicate (in Q) which is not enforced by V, then it must be in the head of the view. (new!)

Example

\[ q(X) : - e(X, Y), e(Y, X), p(X, Y) \]
\[ v_1(U) : - e(U, V), e(V, U) \]
\[ v_2(U, V) : - p(U, V) \]
\[ v_3(U, W) : - e(U, V), e(V, W), p(U, V) \]

\[
\begin{array}{c|c|c|c}
\text{view} & h & \varphi & \text{goals covered} \\
\hline
v_1(U) & U \rightarrow U & X \rightarrow U, Y \rightarrow V & 1, 2? \\
v_2(U, V) & U \rightarrow U, V \rightarrow V & X \rightarrow U, Y \rightarrow V & 3 \\
v_3(U, W) & U \rightarrow U, W \rightarrow U & X \rightarrow U, Y \rightarrow V & 1, 2, 3 \\
\end{array}
\]

\text{C2}
MiniCon Algorithm: Phase 2

**Minimality of MCDs**

Only the **minimal** set of subgoals required to satisfy the Key Property is included in the set of goals $G$ that are covered by MCD.

**Phase 2: combining MCDs**

The only combinations of MCDs that can result in a non-redundant rewriting $s$ of $Q$ are such that:

- the sets of subgoals covered by the MCDs form a partition of the set of subgoals of $Q$. 
MiniCon Algorithm: Phase 2

Running time (worst-case)

The running time of the MiniCon algorithm is $O(nmM^n)$, where

- $n$ is the number of subgoals in the query,
- $m$ is the maximal number of subgoals in a view,
- $M$ is the number of views.
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Back to inverse rules - full dependencies

Extending the inverse rules algorithm
Inverse rule algorithm can be extended to deal with full dependencies and access patterns.
The idea: add new datalog rules to the rewriting.

Rectification
New relation $e$ is added with an intention to capture equality.
Queries should be modified to be able to use $e$.
For example

$q(X) : − \text{pred}(c, X, Y, Y)$

should be rewritten to its rectified version

$\tilde{q}(X) : − \text{pred}(Z, X', Y, Y'), e(X, X'), e(c, Z), e(Y, Y')$
Back to inverse rules - full dependencies

The rules: chase(\(\Delta\))

For each rectified full dependency in \(\Delta\)

\[
\forall \bar{X} \quad p_1(\bar{X}_1) \land \ldots \land p_{n-1}(\bar{X}_{n-1}) \rightarrow p_n(\bar{X}_n)
\]

\((p_i\) are global relations or the relation \(e, \bar{X}_n \subseteq \bar{X},\) and \(\bar{X} = \bar{X}_1, \ldots, \bar{X}_{n-1}\)\)

introduce a new datalog rule in chase(\(\Delta\))

\[
p_n(\bar{X}_n) : - \quad p_1(\bar{X}_1) \land \ldots \land p_{n-1}(\bar{X}_{n-1})
\]

The rewriting

Let \(\Delta\) be a set of full dependencies, \(\forall\) a set of conjunctive source descriptions, and let \(Q\) be a (rectified) query.

Let \(R\) be the set of rules \(\forall^{-1} \cup \text{chase}(\Delta) \cup \text{Equiv}(e)\).

Then \((Q, R)\) is maximally-contained in \(Q\) relative to \(\Delta\).
Access patterns - domain enumeration

Access patterns

\[
\begin{align*}
  s_1^o(X) & : - \text{podsPaper}(X) \\
  s_2^{io}(X, Y) & : - \text{cites}(X, Y) \\
  s_3^i(X) & : - \text{awarded}(X)
\end{align*}
\]

Query

\[
q(X) : - \text{awarded}(X)
\]

The executable rewriting

\[
\begin{align*}
  \text{domain}(X) & : - s_1^o(X) \\
  \text{domain}(X) & : - \text{domain}(Y), s_2^{io}(Y, X) \\
  q(X) & : - \text{domain}(X), s_3^i(X)
\end{align*}
\]
Once again: our setting

We are given:
- a query $Q$ over global schema,
- a set of views with access patterns,
- a set of constraints $\Sigma_c$,

We have to find a query $E$ such that
- $E$ mentions the views literals only,
- $E$ is executable w.r.t. access patterns,
- $E$ is equivalent (or at least minimally-containing) to $Q$ relative to $\Sigma$

where $\Sigma$ contains $\Sigma_c \cup \Sigma_f \cup \Sigma_b$

Forward constraints $\Sigma_f$ and backward constraints $\Sigma_b$

For each $V_i$ in $\mathcal{V}$ we have
- forward constraint: $\forall \bar{X}_i, \bar{Y}_i (\text{body}(V_i) \rightarrow \text{head}(V_i))$
- backward constraint: $\forall \bar{X}_i (\text{head}(V_i) \rightarrow \exists \bar{Y}_i \text{body}(V_i))$
Chase: handling negation

Constraints IC(UCQ\(^{-}\))

\[ \sigma: \forall \bar{X} \ \psi(\bar{x}) \rightarrow \bigvee_{i=1}^{l} \exists \bar{Y}_i \ \xi_i(\bar{X}, \bar{Y}_i) \]

where \( \psi \) and \( \xi_i \) are quantifier-free CQ\(^{-}\)

Step for Q in CQ\(^{-}\)

- Chase step of Q with \( \sigma \) applies iff there is homomorphism \( h \) from \( \psi \) to Q such that for each \( i \), \( h \) has no extension to a homomorphism from \( \psi \land \xi_i \) to Q.
- The result is \( \bigvee_{i=1}^{l} Q \land h'(\xi_i) \) (\( h' \) extends \( h \) to be the identity on \( \bar{Y}_i \))

Negation Constraints \( \Sigma_{\tau}^{-} \)

For each relation \( r \) in the schema \( \tau \), the set \( \Sigma_{\tau}^{-} \) includes the constraint \( \forall \bar{X} \ \text{true} \rightarrow (r(\bar{X}) \lor \neg r(\bar{X})) \)
Rewriting and the chase

ViewRewrite($Q, \Sigma_c, \mathcal{V}$)

1. $Q^1 = \text{chase}(Q, \Sigma_c \cup \Sigma_\tau)$
2. $Q^2 = \text{chase}(Q^1, \Sigma_\mathcal{V}^V \cup \Sigma_\tau^V)$
3. $Q^3 = Q^2|_{\tau_\mathcal{V}}$ (leave the view literals only)
4. $Q^4 = \text{ans}(Q^3)$

If there exists an executable rewriting using views that contains $Q$ (and the chase terminates) then ViewRewrite($Q, \Sigma_c, \mathcal{V}$) returns the minimal executable overestimate of $Q$. 
Is the rewriting equivalent (relative to $\Sigma$)?

**ViewFeasible**($Q$, $\Sigma_c$, $\mathcal{V}$)

1. $Q^4 = \text{ViewRewrite}(Q, \Sigma_c, \mathcal{V})$
2. If $Q^4$ is undefined then return false
3. $Q^5 = \text{chase}(Q^4, \Sigma_b^\mathcal{V} \cup \Sigma^\tau^\mathcal{V})$
4. $Q^6 = \text{chase}(Q^5, \Sigma_c \cup \Sigma^\tau)$
5. $Q^7 = Q^6|_{\tau^\mathcal{V}}$ (drop the view literals)
6. If $Q^7$ is contained in $Q$ return true otherwise return false

If **ViewFeasible**($Q$, $\Sigma_c$, $\mathcal{V}$) terminates then it returns true iff there is an executable rewriting of $Q$ using $\mathcal{V}$ that is equivalent to $Q$ relative to $\Sigma$. 
Conclusions

- Connections of query answering in data integration to incomplete databases as well as to the problem of query containment,
- Inverse rules and MiniCon algorithm that compute maximally-contained rewritings w.r.t conjunctive views,
- Inverse rules algorithm allows for processing recursive queries, full dependencies and access patterns,
- Recursive plans may be necessary when rewriting with access patterns or under functional dependencies,
- Access patterns, constraints and negation can be treated in a uniform way (chase).